Technical Memorandum

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FROM: Dr. M. Mellor, CRREL
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FRACTURE TOUGHNESS MEASUREMENTS FOR ICE

As part of the current sea ice study, CRREL has been asked to comment on fracture toughness measurements for ice and, in particular, to suggest what emphasis should be given to fracture toughness studies.

In responding to the Shell request, we first review some of the relevant terminology and historical developments of the subject.

Toughness. In everyday speech, toughness is a quality which enables people, objects, or materials to endure punishment or strain without yielding. In engineering, toughness is a rather poorly defined concept, but traditionally it has been associated with the capacity of a material to absorb energy before fracturing. Clearly energy alone, as represented by the area under a stress/strain curve, is not an adequate measure of toughness, since high strength and small failure strain could indicate large energy for a very brittle material. Perhaps the best way to define and measure toughness is in terms of the ability to dissipate energy before fracturing. In other words, toughness can be associated with the integral of stress multiplied by inelastic strain, or with total strain energy minus the recoverable strain energy. However, in recent years the term toughness has been appropriated, or perhaps mis-appropriated, for virtually exclusive use within the context of fracture mechanics.

In fracture mechanics the term toughness, or fracture toughness, is defined properly as the critical value of the Irwin parameter G, which is also known as a "crack extension force". G, or G_c, has the dimensions of energy per unit area and it is, in fact, equal to twice the effective specific surface energy for fracture.

However, many practitioners of fracture mechanics refer to the critical stress intensity factor K_c as the fracture toughness of a material, even though K_c has dimensions which have no direct relation to any reasonable definition of toughness. In order to keep things clear, we have to refer back to the origins of fracture mechanics, and to the development of modern notions about fracture toughness.

Griffith Theory. Starting from the observation that the bulk strength of brittle solids is, in general, orders of magnitude lower than the theoretical strength deduced from consideration of interatomic force, A.A. Griffith postulated the existence of minute cracks and associated stress concentrations. Drawing upon the stress analysis given by Inglis for a two-dimensional elliptic crack in an elastic plate, Griffith equated the change of potential energy in the plate to the change of surface energy in the crack as the crack grew in length. For a thin elliptic crack of length 2c:

$$\frac{\partial}{\partial c}\left(\frac{\pi \sigma^2 c^2}{E}\right) = \frac{\partial}{\partial c}\left(4 c \gamma\right) \tag{1}$$

where E is Young's modulus for the plate, γ is the specific surface energy of the material and σ is the applied stress (tensile and perpendicular to the long axis of the crack) at which crack growth occurs. Thus

$$\sigma = \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{E\gamma}{c}\right)^{1/2} \tag{2}$$

for plane stress. For plane strain the corresponding relation is

$$\sigma = \left[\frac{2}{\pi(1-\nu^2)}\right]^{1/2}\left(\frac{E\gamma}{c}\right)^{1/2} \tag{3}$$

Where ν is Poisson's ratio. Numerically, the two equations are not much different. Much the same result is obtained by direct consideration of theoretical material strength and stress concentration at the end of an elliptic crack. From consideration of interatomic forces as a function of separation, the theoretical tensile strength of the material σ_* is

$$\sigma_* = \left(\frac{E\gamma}{a}\right)^{1/2} \tag{4}$$

where a is the atomic spacing in the unstrained state. From the Inglis stress analysis for an elliptic crack with tip radius ρ, the stress at the crack tip σ_{ct} is

$$\sigma_{ct} = 2\sigma\left(\frac{c}{\rho}\right)^{1/2} \tag{5}$$

where σ is the applied stress in the plate. Equating σ_{ct} to σ_* for crack growth:

$$\sigma = \left(\frac{\rho}{4a}\right)^{1/2}\left(\frac{E\gamma}{c}\right)^{1/2} \tag{6}$$

in which ρ is considered to be of the same order of magnitude as a for a sharp crack. This version is identical to Griffith's plane stress relation if

$$\rho = (8/\pi)\, a = 2.55\, a \tag{7}$$

In addition to providing a reasonable physical explanation for the discrepancy between theoretical strength and actual strength, Griffith was able to develop a failure criterion for the onset of brittle fracture in multiaxial stress states.

Modification of Griffith Theory. Griffith developed his theory primarily to explain the properties of glass, and the theory was later believed to be generally applicable to brittle solids. However, if the strength equations which contain the surface energy γ are applied to metals or polymers, the predicted strength often turns out to be very much lower than the actual strength of the real material. This can be explained by plastic yielding at critical stress concentrations, which has the effect of blunting the cracks.

In the late forties, Orowan and Irwin independently modified the Griffith equation for strength by taking into account the energy dissipated in localized plastic yielding, while at the same time retaining the elastic analysis for the overall effect of a crack because the plastic yield zones were considered small relative to the crack length. Orowan substituted for the surface energy γ a term which included a specific energy for plastic working γ_p:

$$\sigma = \left(\frac{2}{\pi}\right)^{1/2}\left[\frac{E(\gamma+\gamma_p)}{c}\right]^{1/2} \approx \left(\frac{2}{\pi}\right)^{1/2}\left[\frac{E\gamma_p}{c}\right]^{1/2} \tag{8}$$

for plane stress. The approximation follows from the fact that $\gamma_p \gg \gamma$. Irwin expressed the same idea by denoting the critical rate of change of energy with crack length by a parameter G_c. Being an energy per unit area, G_c has the dimensions of force per unit length, and it is referred to as a crack extension force. In the Irwin formulation

$$\sigma = \left(\frac{1}{\pi}\right)^{1/2}\left(\frac{EG_c}{c}\right)^{1/2} \tag{9}$$

for plane stress. Thus the Orowan and Irwin expressions are identical with

$$G_c = 2(\gamma + \gamma_p) \tag{10}$$

An important feature of the Irwin and Orowan modifications is the combination of elastic and plastic assumptions. The local stress field near a crack tip is allowed to create plastic yielding, but the overall solid matrix is still assumed to behave elastically. Obviously, these assumptions can only be justified if: (1) the spacing between cracks is significantly greater than the extent of the plastic yield zones at the crack tips, and (2) <u>the solid matrix really is elastic</u>.

<u>Fracture Mechanics and Fracture Toughness</u>. The name "fracture mechanics" has come to be used, somewhat restrictively, for study of the effect of cracks on the bulk strength of solid materials. It derives from Griffith theory, and from the later modifications of that theory by Irwin and Orowan, as outlined above.

Griffith's original idea was that fracture occurred when a crack extended without limit because an increment of crack extension involved a gain of surface energy U_s less than the drop of potential energy of the surrounding elastic material U_p:

$$|\delta U_p| > |\delta U_s| \tag{11}$$

Irwin and Orowan introduced the idea of energy dissipation by plastic yielding at a crack tip (δW_p) and the possibility of external work input to the system (δW_e), making the critical energy balance:

$$\delta U_p + \delta W_e \geqslant \delta U_s + \delta W_p \tag{12}$$

Since $\delta W_p \gg \delta U_s$ and δW_e is, for all practical purposes, zero, the condition simplified to

$$\delta U_p \geqslant \delta W_p \tag{13}$$

The change of potential energy δU_p as the crack extends by an increment of length δx can be equated to a unit force G multiplied by δx :

$$\delta U_p = G \, \delta x \tag{14}$$

or

$$G = \frac{\delta U_p}{\delta x} \tag{15}$$

This is Irwin's crack extension force, which was mentioned earlier. From elastic analysis, the critical value G_c for unstable crack extension is

$$G_c = \frac{\pi c}{E} \cdot \sigma^2 \tag{16}$$

for plane stress, and

$$G_c = \frac{\pi (1 - \nu^2) c}{E} \sigma^2 \tag{17}$$

for plane strain, where σ is the applied stress at failure.

Analysis of the stress distribution around an idealized crack in an elastic plate gives stress fields that are geometrically similar for geometrically similar "cracks". The absolute magnitude of a given stress component is proportional to the stress applied to the plate, σ, and it is also proportional to the square root of a characteristic linear dimension of the crack, such as the half-length of the major axis c. Thus the effects of geometric scale and stress level can be expressed by a <u>stress intensity factor K</u> which contains the product $\sigma \times \sqrt{c}$. For convenience, K is defined as

$$K = \sigma (\pi c)^{1/2} \tag{18}$$

This is obviously another way of expressing the crack extension force G. In terms of the critical values for failure, K_c and G_c:

$$G_c = \frac{K_c^2}{E} \tag{19}$$

for plane stress, and

$$G_c = \frac{K_c^2 (1 - \nu^2)}{E} \tag{20}$$

for plane strain.

In this summary of crack analyses the basic ideas have been developed with reference to the opening or closing of a two dimensional crack in a plate that is under uniaxial tension or compression. However, in fracture mechanics three distinct types of crack motion are recognized (Fig. 1). Mode I is the simple separation considered for the foregoing discussion. Mode II is in-plane shearing displacement, with opposite faces of a flat crack sliding across each other in the direction of the crack's major axis. Mode III involves twisting, and sliding of opposing crack faces in a direction normal to both axes of the two-dimensional crack. As far as materials testing is concerned, interest centers on Mode I, and virtually all test methods are designed to extend cracks according to Mode I. The critical value of the stress intensity factor for Mode I is denoted by the symbol K_{Ic}.

Fig. 1 Displacement and crack propagation modes

Application of Griffith Theory to Ice. There is no reason to believe that basic Griffith theory will have much relevance to the failure of ice at low strain rates. However, for high strain rates ($\geq 10^{-3}$ s^{-1} at typical temperatures) there is ample evidence that ice deforms elastically, with a modulus close to the true elastic modulus. Thus before applying modern fracture mechanics, which was developed largely to explain the inapplicability of Griffith theory for certain materials, we should check to see whether Griffith theory might apply to ice under high strain rates.

Equation (2) provides a means of calculating the uniaxial tensile strength of ice σ_T as a function of the controlling flaw size when Young's modulus E and the specific surface energy γ are known. For ice of very low porosity, ($\leq$ 0.01), the true Young's modulus E is 9 to 10 GPa at typical temperatures. For non-saline ice, the vapour/solid specific energy γ is approximately 0.1 J/m^2, and the liquid/solid surface energy is about 30% of the vapour/solid value (Fletcher, 1970; Hobbs, 1974.* The vapour/solid value is probably the appropriate one

*Liu and Miller (1979) use values that are off by two orders of magnitude due to incorrect conversion of Fletcher's values.

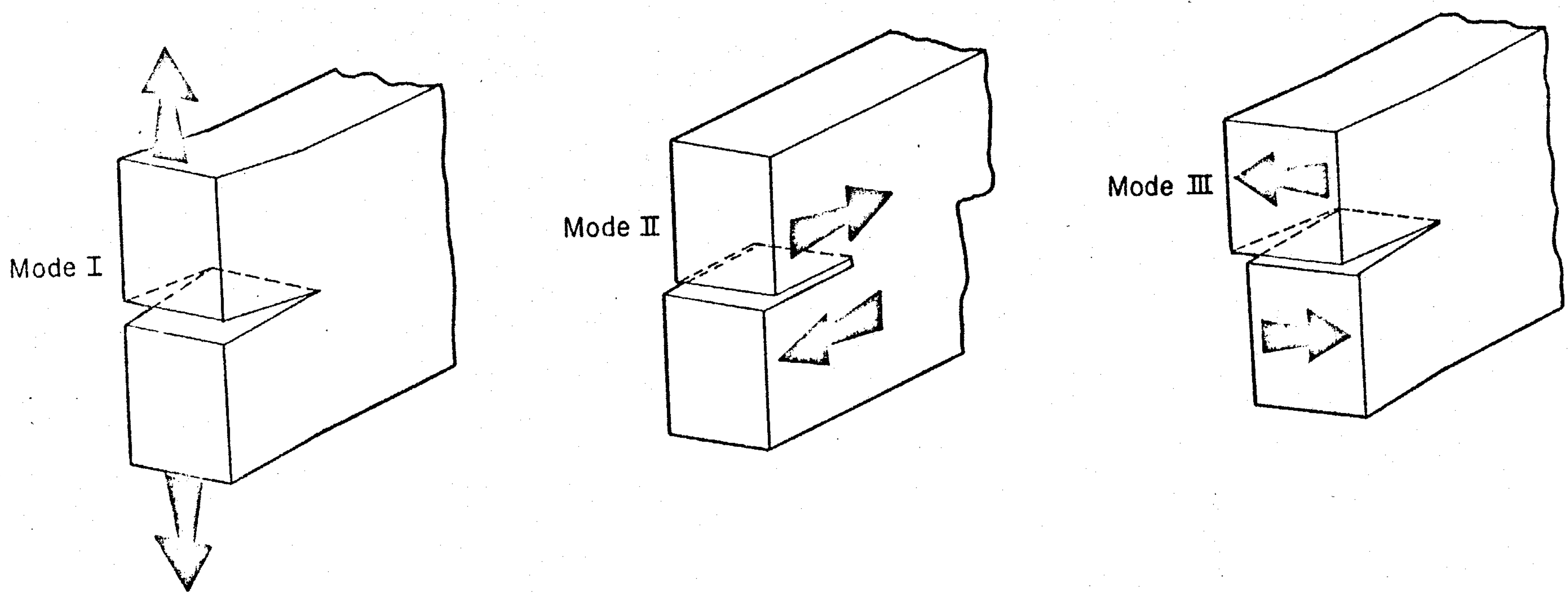

Fig.1. Displacement and crack propagation modes

for consideration of brittle fracture in "cold" ice, but the lower liquid/solid value might be applicable in ice which has a "liquid-like layer" or liquid-filled flaws. The latter condition might give something equivalent to the Rehbinder, or Joffe, effect, whereby γ is reduced by adsorption of certain surface-active chemicals and strength decreases in consequence.

If we substitute into equation (2) $E = 10\ GPa$ and $\gamma = 0.1\ J/m^2$,

$$\sigma_T = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{10^{10} \times 10^{-1}}{c}\right)^{1/2}$$
$$= \frac{2.52 \times 10^4}{\sqrt{c}}\ Pa$$

where the half-length of the controlling flaw, c, is in metres. In figure 2, the resulting calculated values of σ_T are given as a function of the flaw size 2c.

Figure 2 gives a comparison of calculated values with measured values of for non-saline ice, making certain assumptions about "flaw size" for the various test specimens. In none of the test specimens were Griffith cracks actually observed or measured, and so identifiable structural dimensions such as grain size and bubble size have been used to permit plotting of the data. It seems unlikely that the controlling "Griffith crack" could be larger than the grain size in these intact lab specimens, but it is conceivable that the controlling flaws could be smaller than the grain diameter, perhaps by a factor of 2 or 3 if we are considering a mosaic of equant but angular grains.

The real importance of figure 2 is that it gives theoretical strength values which are credible in comparisons with actual test data. While figure 2 does not prove that simple Griffith theory is valid for ice, it certainly gives little reason for rejecting Griffith theory out of hand. In other words, we have no need to invoke a specific energy for plastic working (γ_P) which is orders of magnitude greater than γ, as is apparently the case for metals.

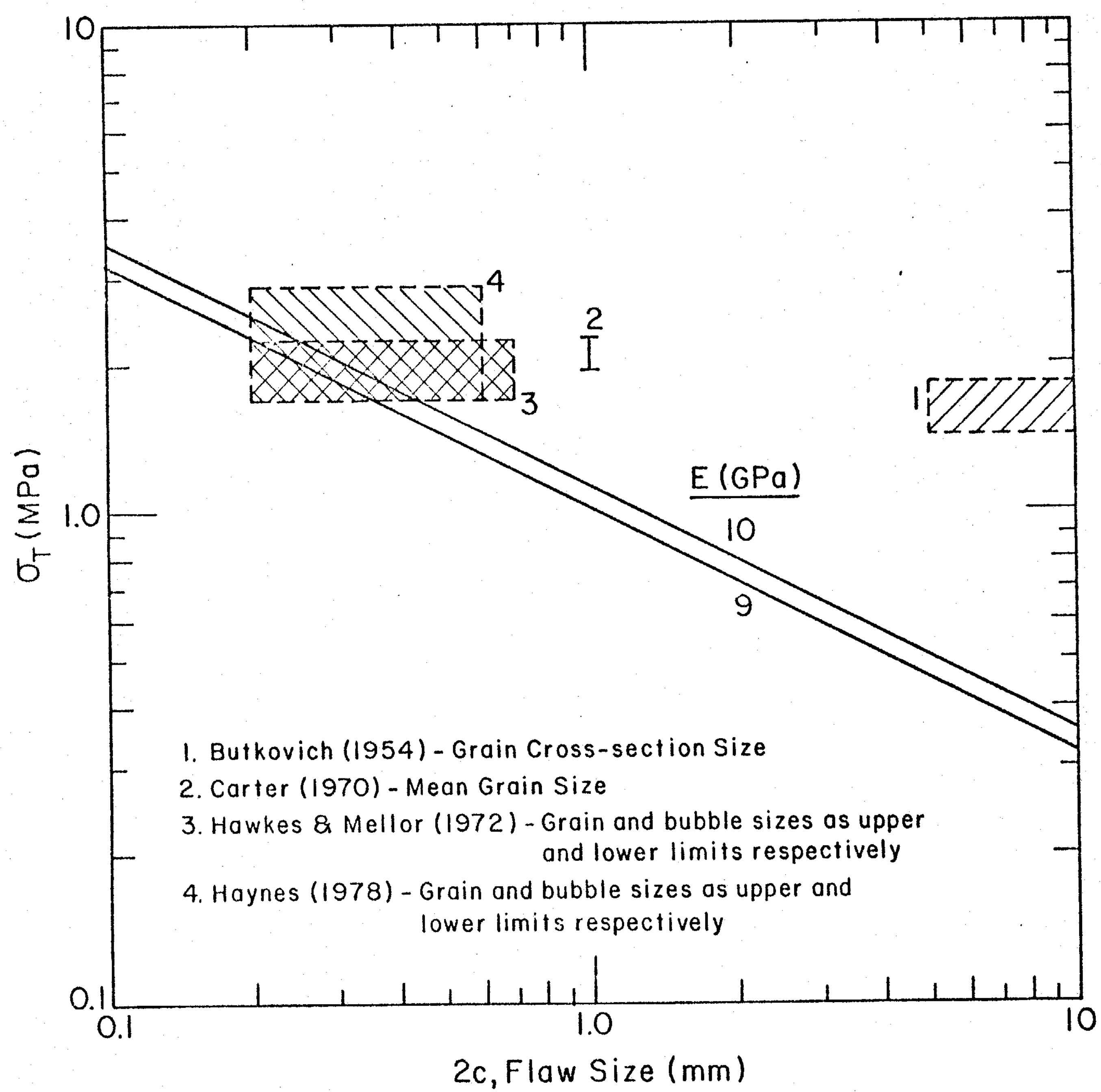

Fig 2 Comparison of theoretical tensile strength with measured values.

If simple Griffith theory were to prove valid for ice, there would be little justification for studying fracture toughness, which is a measure of a material's departure from simple Griffith behaviour. However, various investigations have measured fracture toughness, and it is necessary to review the data.

Fracture toughness of ice. Virtually all fracture toughness measurements on ice depend on tests which flex or pry open a crack in "Mode I". Test data are thus presented in terms of the critical stress intensity factor, K_{Ic}. Because the measured values for K_{Ic} vary greatly, and because we need some intuitive "feel" when considering these values, it is worth recalling what K_{Ic} means.

Toughness is measured by the specific energy dissipation at failure G_c, which is also known as the critical crack extension force. K_{Ic} is related to G_c by

$$K_{Ic} = (E G_c)^{1/2}$$

in plane stress, and

$$K_{Ic} = \left(\frac{E G_c}{1-\nu^2}\right)^{1/2}$$

in plane strain. Thus there is a simple direct relation between K_{Ic} and G_c if E is a constant. K_{Ic} is also related to the overall tensile failure stress of the material σ :

$$K_{Ic} = \sigma (\pi c)^{1/2}$$

where c is the half-length of the controlling cracks. This relation implies that, if c is constant and the stress state does not change, K_{Ic} is directly proportional to the bulk strength of the material.

For ice straining at low rates, say less than $10^{-6}\ s^{-1}$, we would not expect K_{Ic} to have any relevance, since the ice is inelastic and it flows without cracking.

At extremely high rates and low temperatures, ice could conceivably become perfectly elastic and perfectly brittle, and under such conditions the original Griffith theory ought to apply. For such a limit, with γ_p tending to the specific surface energy γ, K_{Ic} would tend to a low value:

$$K_{Ic} \rightarrow (2E\gamma)^{1/2}$$

in plane stress. Taking E = 10 GPa and the grain boundary specific energy $\gamma = 0.1$ J/m^2 for freshwater ice, the lower limit of K_{Ic} might be about 45 kN-m$^{-3/2}$ for plane stress.

Measured values of K_{Ic} for ice are typically of order 100 kN-m$^{-3/2}$. This is not much higher than the "Griffith" value, and it implies that $\gamma_p \approx 5\gamma$, assuming that E is more or less constant.

When strain rate, or loading rate, is varied in a fracture toughness test for a given type of ice, we would expect K_{Ic} to decrease as $\dot{\varepsilon}$ or $\dot{\sigma}$ increases, at least for non-saline ice. While at least one set of experiments shows a trend opposite to this, the overall trend shown by compilation of published data is in the expected sense (Fig. 3, 4). Rate effects were originally expressed in terms of the speed of the testing machine, which is clearly of limited interest, but now the accepted rate variable seems to be $\dot{K}_I$, which is really the inverse of the time to failure. Strain rate has been used as a variable, but there are some problems of interpretation.

In sea ice, K_{Ic} has been found to decrease with increase of loading rate for $\dot{K}_I > 10^{-2}$ kN-m$^{-3/2}$ s^{-1}, or effective $\dot{\varepsilon} > 10^{-3}$ s^{-1} (Urabe et al., 1980; Urabe and Yoshitake, 1981a & b). However, for lower rates K_{Ic} appears to be insensitive to rate (Fig. 5). The lowest measured values for sea ice are lower than the expected "Griffith value" for pure ice.

In discussing rate effects, we have assumed that K_{Ic} will decrease as the material become more elastic and more brittle due to higher loading rates. Extending this line of argument to temperature effects, we might therefore expect K_{Ic} to decrease as temperature decreases, since lower temperature undoubtedly makes ice more elastic and more brittle. However, experimental data (Fig. 6) seem to show exactly the opposite trend, with K_{Ic} increasing as temperature decreases. This observed trend is consistent with the fact that tensile strength σ_T increases as temperature decreases, since K_{Ic} is proportional to strength if the crack length 2c is constant. Nevertheless, there does appear to be fundamental contradiction between the observed temperature effect and the rate effect if the ideas of fracture mechanics are applicable to ice.

If measurements of K_{Ic} are valid, they permit a systematic treatment of flaw size. For a constant value of K_{Ic} and variation of crack length 2c between samples, the tensile strength σ_T might be expected to be inversely proportional to $\sqrt{c}$. Urabe and Yoshitake (1981) tested both notched and un-notched beams with varying grain size in order to calculate flaw size for the ice, and they found a perfect 1:1 correlation between calculated flaw size and observed grain size. However, this experiment appears to merit further discussion, since both σ_{FT} and K_{Ic} were functions of grain size, and the effect of grain size on σ_{FT} appears to be in the wrong direction.

Vaudrey (1977) measured K_{Ic} for sea ice at -10° and $-20^{\circ}C$, and plotted the results against the square root of brine volume for a very narrow range. Vaudrey drew a line on the graph to indicate linear decreas of K_{Ic} with increase in the root of brine volume but, in fact, there was no significant correlation between the variables (K_{Ic} values scattered by a factor of 5). Shapiro et al (1981) made measurements in the same range (brine porosity 0.16 to 0.38), and showed a more convincing decrease of K_{Ic} with increase of porosity, although there was still large scatter.

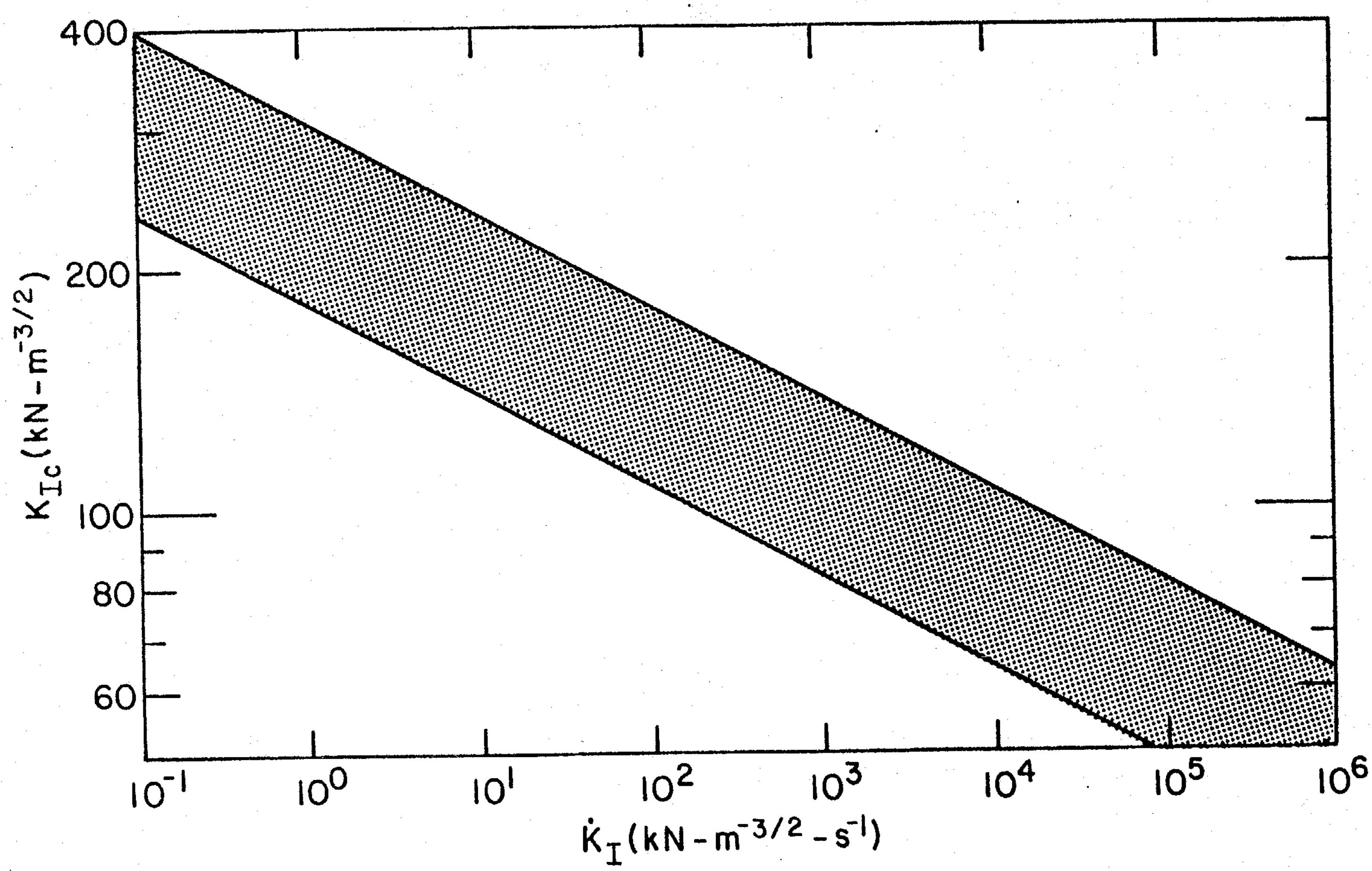

Fig 3. Effect of loading rate on K_{IC} for non-saline ice (from data summarized by Urabe and Yoshitake, 1981).

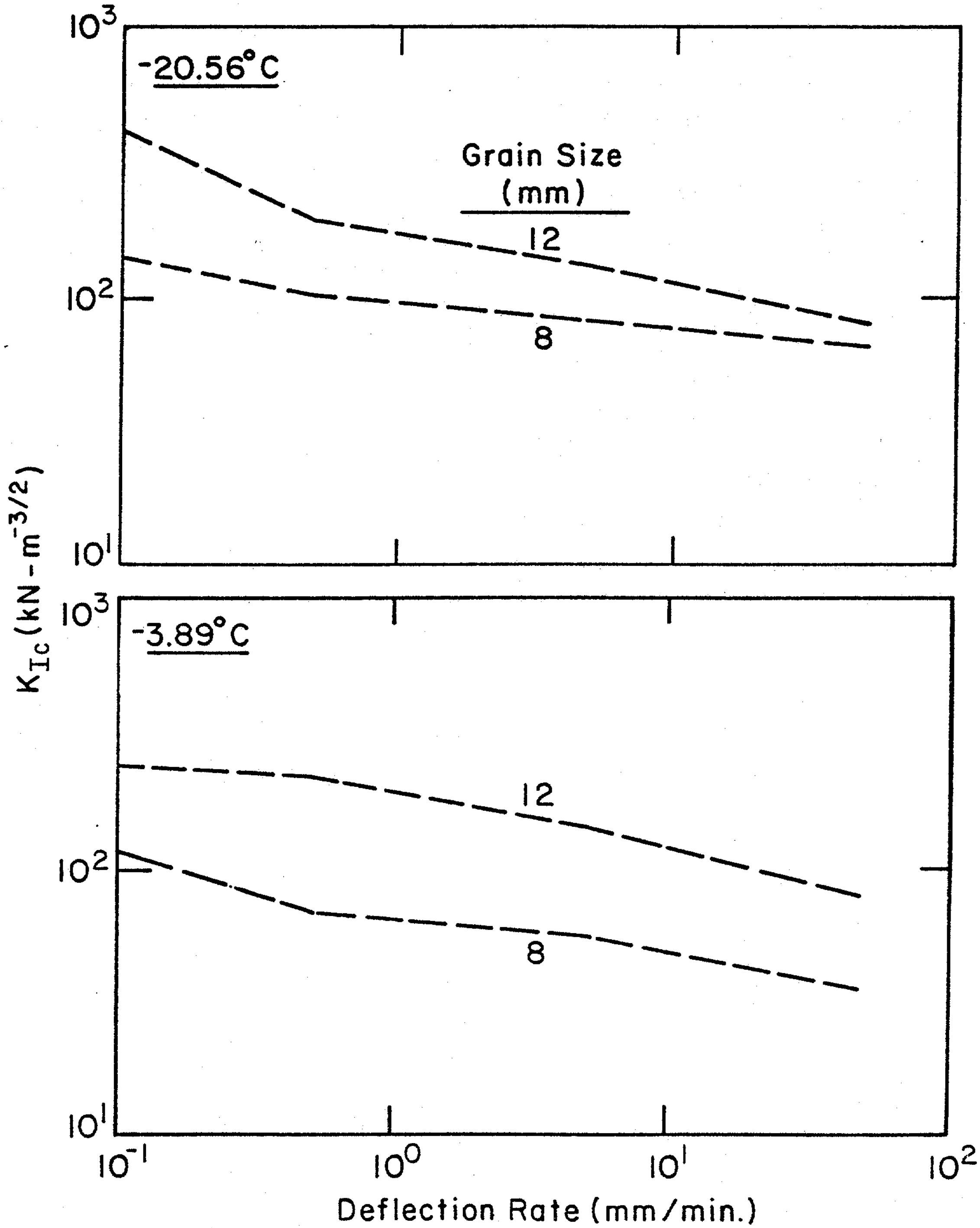

Fig. 4 Variation of K_{IC} with loading rate, grain size and temperature for columnar freshwater ice (Hamza and Muggeridge, 1979).

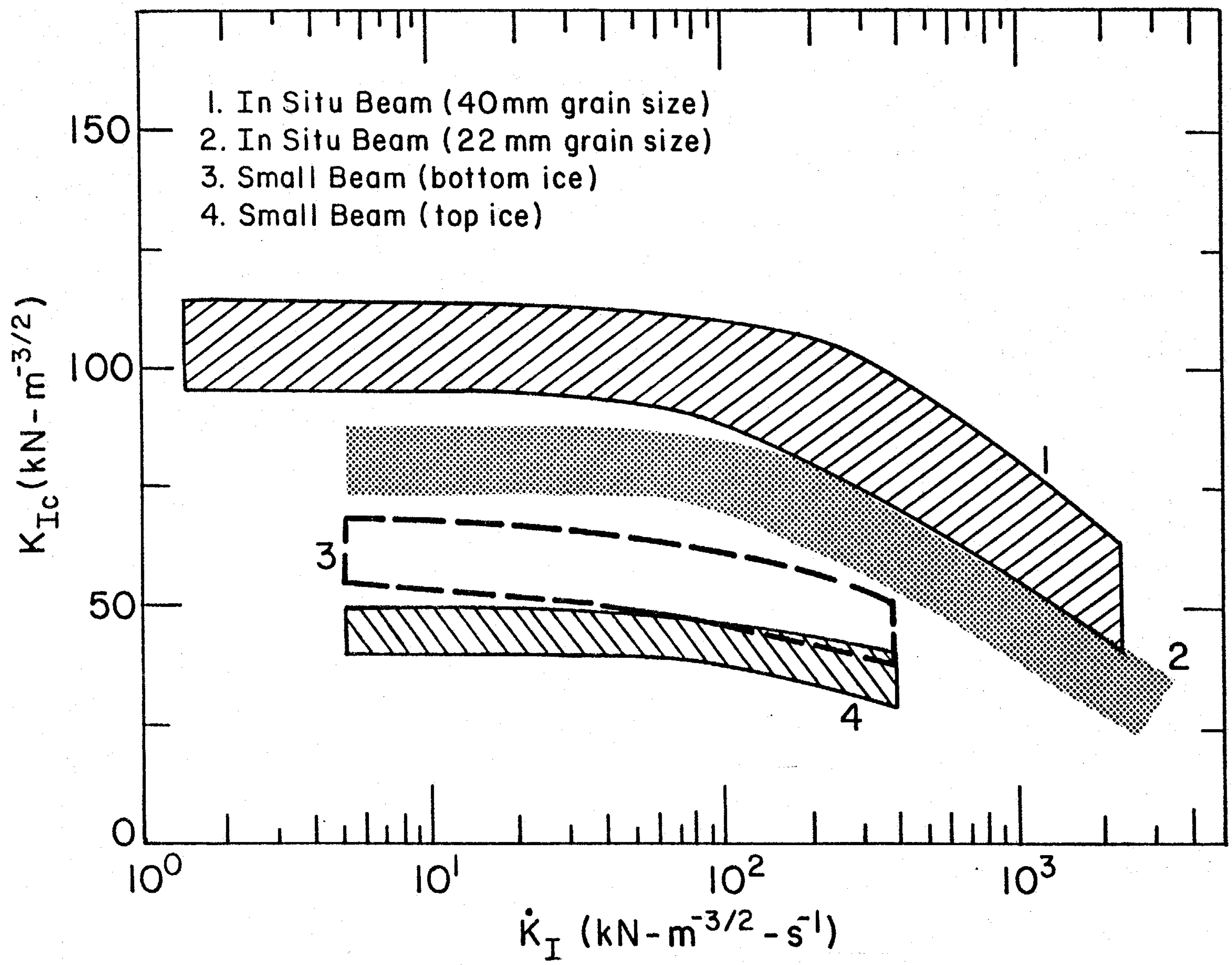

Fig 5 Effect of loading rate on K_{IC} for sea ice (data from Urabe and Yoshitake, 1981).

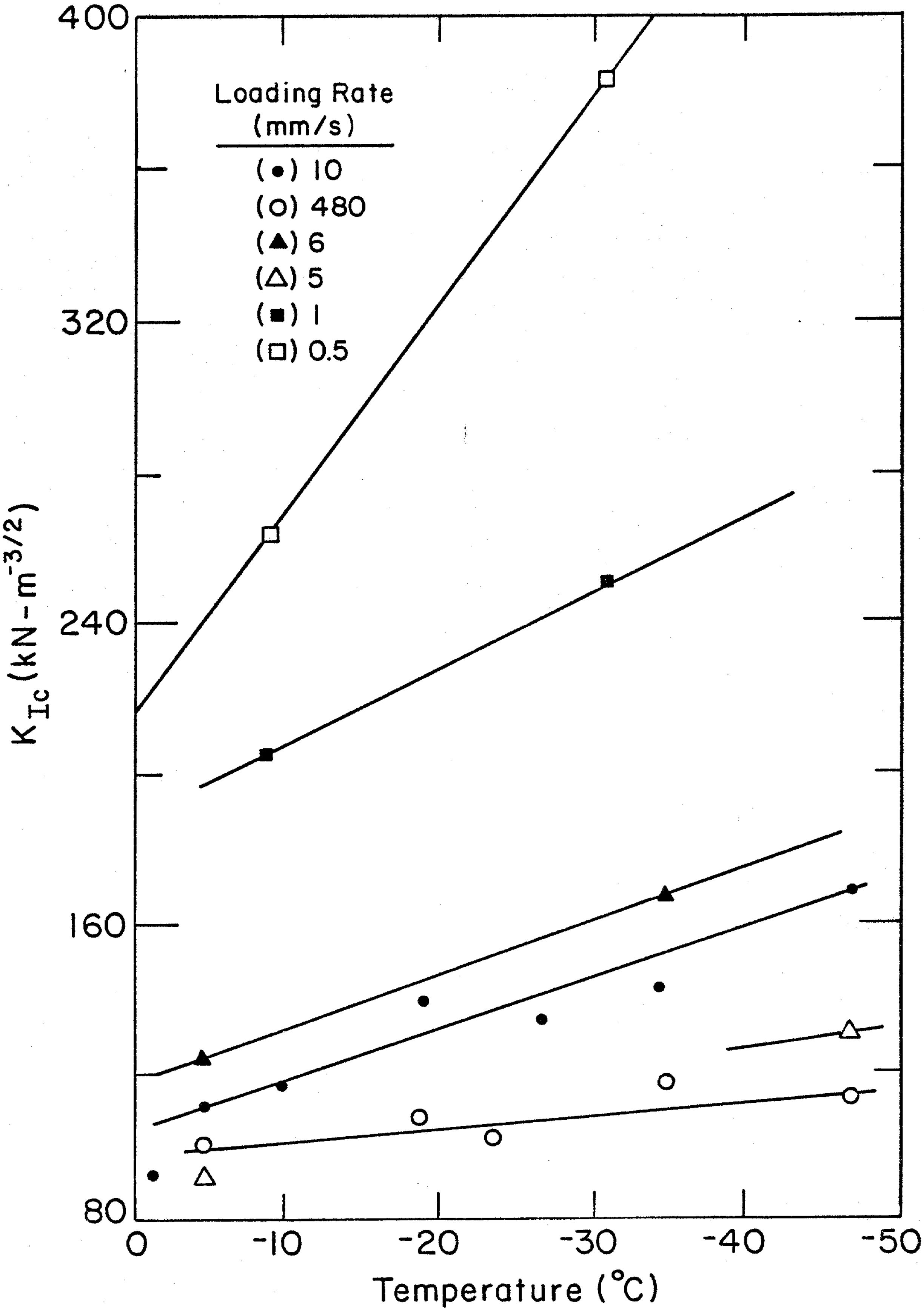

Fig 6 Variation of K_{IC} with temperature and loading rate (from Miller, 1980).

Discussion. When ice is tested at high rates, so that its behaviour is almost purely elastic, simple Griffith theory gives credible predictions of tensile strength, and it predicts values of K_{Ic} which are very close to the values measured at high strain rates.

If ice behaves perfectly elastically, we expect little variation of σ_T and K_{Ic} with temperature, since decrease of temperature involves a slow increase of $\dot{E}$ and a slow decrease of γ. The experimental data for σ_T are consistent with this speech and the data for K_{Ic} given by Miller (fig 6) suggest that the expectation might be borne out if loading rates are sufficiently high.

At the same time, purely elastic behaviour ought to eliminate variation of σ_T and K_{Ic} with strain rate, since neither E nor γ are expected to be significantly rate-dependent. The limited data for σ_T as a function of $\dot{\varepsilon}$ (Hawkes and Mellor, 1972) are consistent with this expectation, but data for K_{Ic} do not appear to be tending to a limiting value for high strain rates.

To sum up the foregoing, ice loaded at high rates behaves elastically, and the limited experimental data for σ_T and K_{Ic} at high rates are not in serious conflict with the predictions of Griffith theory.

Going to the other extreme of behaviour, when ice is strained at very low rates its elastic behaviour is completely overwhelmed by non-linear viscous flow. In this range of behaviour there is no justification for applying elastic fracture mechanics, and K_{Ic} has no significance whatsoever.

This leaves the question of the intermediate range, where elastic deformation and viscous flow both contribute significantly to the total strain. In considering the possible relevance of K_{Ic} for this range, it is important to keep in mind the derivation of the relevant theory, and also the distinction between ideal plasticity and viscous flow.

In deriving the theoretical framework into which K_{Ic} fits, it is assumed that the solid material is elastic-plastic, so that the general matrix can remain elastic while only the most highly stressed zones suffer plastic yielding. However, ice does not have a finite yield stress; it begins to flow at very low stresses, and the flow rate increases with the third or fourth power of stress. Thus, if loading rate is low enough to permit significant inelastic strain prior to final failure, it is unlikely that elastic-plastic fracture mechanics would be applicable. Nevertheless, there might be a range of behaviour, at rates just below those which give purely elastic response, where K_{Ic} is a useful parameter. To examine this possibility, we have to reconsider the experimental data.

K_{Ic} is supposed to be a measure of toughness, and a material's ability to resist weakening by flaws and stress concentrations. We therefore expect K_{Ic} to increase as ductility increases, but we have to keep in mind that increase of K_{Ic} would usually be reflected by an increase of strength.

For ice, K_{Ic} certainly appears to increase as strain rate decreases from the pure elastic range. There are also experimental data showing increase of strength as strain rate decreases through the same range, but these data are not yet conclusive because of the possibility that the trend is caused by imperfections in test technique.

However, as ice temperature decreases, the limited data show K_{Ic} <u>increasing</u>, even though the material is undoubtedly becoming more elastic and more brittle. This trend of K_{Ic} corresponds to the trend shown by strength, indicating a degree of internal consistency in the theoretical ideas, but it is in direct conflict with the strain rate response. Perhaps more to the point, it is in conflict with common sense --- ice does not get tougher as it gets colder.

The reasons for this strange behaviour of K_{Ic} are not immediately obvious, but one might suspect the test method, which is usually beam flexure. Overall, the strength data from beam flexure tests on ice are wildly inconsistent, and it is not hard to see why. The basic assumptions for beam analysis are as follows:

(1) linearly elastic homogeneous material, (2) equal moduli in tension and compression, (3) small strains, with cross-sections remaining plane and mutually parallel. These are met only at very high strain rates, where the test becomes very sensitive to imperfections of specimen preparation and loading technique. Even if a perfect test is made at high rate, fracture initiates at the surface, the zone of critical stress is very thin, and the crack propagates in a stress gradient. If conditions are such that the beam is not perfectly elastic, the degree to which the assumption remain valid varies with temperature and strain rate. Thus the variation of "flexural strength" with strain rate and temperature is unlikely to provide a good indication of the variation of σ_T with strain rate and temperature. When ice beams are notched for fracture toughness tests, a further level of complication is introduced.

Conclusions. When ice behaves elastically ($\dot{\varepsilon} \geq 10^{-3}\ s^{-1}$ at typical temperatures), simple Griffith theory can be used to assess the effects of flaws and stress concentrations.

When ice is subject to significant creep ($\dot{\varepsilon} \leq 10^{-5}\ s^{-1}$ at typical temperatures), K_{Ic} has no significance.

For the range of behaviour where ice is quasi-brittle (say 10^{-5} to $10^{-3}\ s^{-1}$ at typical temperatures), the existing data for K_{Ic} are not easy to accept at face value. Until the apparent contradictions are resolved, it would seem unwise to use K_{Ic} as a design parameter.

Recommendations. Fracture toughness measurements by other research groups should be kept under review. New measurements by Shell probably ought to be deferred until beam flexure testing has been subjected to critical examination. It may be necessary to devise new test methods in order to obtain reliable measurements of K_{Ic} for ice.

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Appendix

Dislocation theory for crack nucleation

Griffith theory and its derivatives deal with the growth of existing cracks. There is another body of theory which deals with the nucleation of cracks by pile-up of dislocations. The latter theory is usually considered to have originated with Zener, and its development is associated with the names Stroh, Petch, Cottrell, Smith and Barnby.

The dislocation theory gives an expression for the effective shear stress τ_{eff} which is needed to produce crack nucleation:

$$\tau_{eff} = \left(\frac{3\pi \gamma G}{8(1-\nu) L} \right)^{1/2} \qquad \text{(A-1)}$$

where G is the shear modulus and L is the length of the dislocation pile-up. Substituting for G in terms of E and rearranging the equation:

$$\tau_{eff} = \left(\frac{3\pi}{16(1-\nu^2)} \right)^{1/2} \left(\frac{E\gamma}{L} \right)^{1/2} \qquad \text{(A-2)}$$

This is similar in form to the Griffith equation if L is thought of as a flaw size. However, τ_{eff} is the shear yield stress τ_y, which is directly proportional to σ_T, minus a "friction stress" τ_i which resists dislocation notion:

$$\tau_{eff} = \tau_y - \tau_i \qquad \text{(A-3)}$$

Carter (1970) applied these ideas to ice, taking $\tau_y = \sigma_T/2$ and L = d/2, where d is the grain diameter (others have taken L = d). He apparently determined τ_i experimentally as 3 kgf/cm^2, but did not explain how this was done. The prediction equation was thus

$$\sigma_T = \left(\frac{3\pi}{2(1-\nu^2)} \right)^{1/2} \left(\frac{E\gamma}{d} \right)^{1/2} + 2\tau_i \qquad \text{(A-4)}$$

This gives a grain size dependence similar to that predicted by the Hall-Petch relation, instead of the simple $d^{-\frac{1}{2}}$ relation which is obtained by identifying grain size with Griffith cracks.